Ch. 18

4. $z = \frac{\omega_0}{\omega_r} - 1 = \frac{a(t_r)}{a_0} - 1$

ω_0 = frequency emitted today
ω_r = received frequency
t_r = age of universe at reception

$t_r = t_0 + 10\ \text{Gyr}$

For a flat, matter-dominated universe

$a(t) = \left(\frac{t}{t_0}\right)^{2/3}$ & $t_0 = \frac{2}{3} t_H$ using $a_0 = 1$ to set the scale

$z = \left(\frac{t_0 + 10\ \text{Gyr}}{t_0}\right)^{2/3} - 1$

$t_0 = \frac{2}{3}\, 9.78\, h^{-1}\ \text{Gyr}$ $h = 0.72$
$= 9.06\ \text{Gyr}$

$z = 0.64$

OR $\omega_r = 0.61\, \omega_0$

6. $\frac{\delta t_e}{\delta t_0} = \frac{a(t_e)}{a_0} = \frac{1}{1+z} = \frac{1}{2.1}$ $\delta t_0 = 2$ mo.

$\delta t_e = 0.95$ mo.

13. From eq. (18.69) we have

$a(\eta) = \frac{\Omega}{2H_0(\Omega - 1)^{3/2}}(1 - \cos\eta)$ for a closed, matter-dominated universe

$t(\eta) = \frac{\Omega}{2H_0(\Omega - 1)^{3/2}}(\eta - \sin\eta)$

Maximum volume occurs at $\eta = \pi$

$a(\pi) = \frac{\Omega}{H_0(\Omega - 1)^{3/2}} = a_{max}$

$V_{max} = 2\pi^2 a_{max}^3$ (from eq. 18.70)

$= 10^{12}\ \text{Mpc}^3$

$\Rightarrow a_{max} = 3{,}700\ \text{Mpc}$