Ch. 9

2. $e^- + e^+ \rightarrow 2\gamma$

Each γ will have energy $E_* = m_e c^2 \approx 0.5\,\text{MeV}$

$$E_\infty = E_*\left(1 - \frac{2GM}{c^2 R}\right)^{1/2}$$

$$= 0.5\,\text{MeV}\left[1 - \frac{2(6.67\times10^{-8}\,\frac{\text{cm}^3}{\text{s}^2\text{g}})(2.5)(1.99\times10^{33}\,\text{g})}{(3\times10^{10}\,\frac{\text{cm}}{\text{s}})^2(10^6\,\text{cm})}\right]^{1/2}$$

$$= 0.26\,\text{MeV}$$

3. a. Normalization of 4-momentum gives

$$\mathbf{p}\cdot\mathbf{p} = -m^2$$

We also have

$$E = -\mathbf{p}\cdot\mathbf{u}_{obs}$$

In orthonormal basis of observer

$$\mathbf{p}\cdot\mathbf{p} = -(p^{\hat{0}})^2 + |\vec{p}|^2 = -m^2 \quad \& \quad E = -\mathbf{p}\cdot\mathbf{e}_{\hat{0}} = p^{\hat{0}}$$

$\therefore\ E^2 = m^2 + |\vec{p}|^2$ just as in special relativity

b. Stationary observer: $\mathbf{u}_{obs} = (u^t_{obs}, 0, 0, 0)$

Normalization of 4-velocity gives

$$\mathbf{u}_{obs}\cdot\mathbf{u}_{obs} = g_{\alpha\beta}\, u^\alpha_{obs}\, u^\beta_{obs} = -\left(1 - \frac{2M}{r}\right)(u^t_{obs})^2 = -1$$

$$\Rightarrow u^t_{obs} = \left(1 - \frac{2M}{r}\right)^{-1/2}$$

$$E = \left(1 - \frac{2M}{r}\right) p^t\, u^t_{obs}$$

$$\Rightarrow p^t = E\left(1 - \frac{2M}{r}\right)^{-1/2}$$

$|\vec{p}| = \mathbf{p}\cdot\mathbf{e}_{\hat{r}}$ Assuming only radial motion

orthogonality

$$\mathbf{e}_{\hat{0}}\cdot\mathbf{e}_{\hat{r}} = -\left(1 - \frac{2M}{r}\right)\left(1 - \frac{2M}{r}\right)^{-1/2} e^t_{\hat{r}} = 0$$

$$\Rightarrow e^t_{\hat{r}} = 0$$