Ch. 7

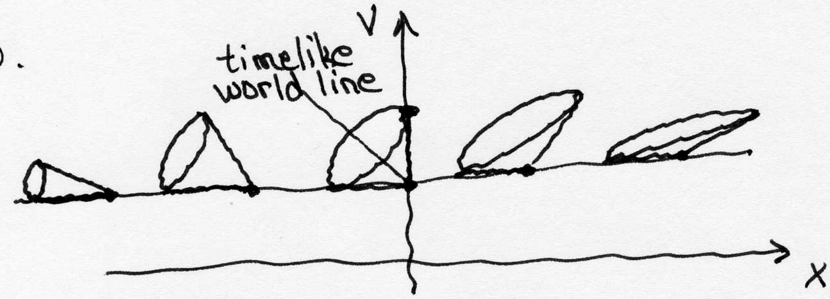
5. $ds^2 = -x\,dv^2 + 2\,dv\,dx$

a. Null separation $ds^2 = 0$ $\quad \dfrac{dv^2}{dv\,dx} = \dfrac{2}{x}$

Edges of light cones are curves w/

$$\frac{dv}{dx} = 0 \quad \& \quad \frac{dv}{dx} = \frac{2}{x}$$

b.



Timelike separation requires $ds^2 < 0$. For this to be true at $x=0$, the future light cone must tilt to the left

c. The timelike world line above shows that a particle can move from positive x to negative x. No timelike worldline passes through $x=0$ from negative to positive.

7. b. $\quad ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta \qquad dx^\alpha = \dfrac{\partial x^\alpha}{\partial x'^\gamma} dx'^\gamma$

$$ds^2 = g_{\alpha\beta} \frac{\partial x^\alpha}{\partial x'^\gamma} dx'^\gamma \frac{\partial x^\beta}{\partial x'^\delta} dx'^\delta$$

$$= g_{\alpha\beta} \frac{\partial x^\alpha}{\partial x'^\gamma} \frac{\partial x^\beta}{\partial x'^\delta} dx'^\gamma dx'^\delta$$

$$= g'_{\gamma\delta}\, dx'^\gamma dx'^\delta$$ from definition of the metric

$$\therefore \; g'_{\gamma\delta} = g_{\alpha\beta} \frac{\partial x^\alpha}{\partial x'^\gamma} \frac{\partial x^\beta}{\partial x'^\delta}$$