1. a. $t = \frac{D}{\sigma_r} = \frac{(6\times10^6 \text{ pc})\left(\frac{3.09\times10^{16}\text{ m}}{\text{pc}}\right)}{9.77\times10^5 \text{ m/s}}$

$= 1.90\times10^{17}\text{ s} = 6.01\times10^9 \text{ yr}$

b. $t_H = \frac{1}{H_0} = \frac{1}{(3.24\times10^{-18})(0.71)}\text{ s}$

$= 4.22\times10^{17}\text{ s} = 1.33\times10^{10}\text{ yr}$

c. Must be gravitationally bound. Have had plenty of time to escape cluster otherwise.

2. a. $M = 10^8 M_\odot$ $\quad r = R_S = \frac{2GM}{c^2} = \frac{2(6.67\times10^{-11}\frac{\text{m}^3}{\text{kg s}^2})(10^8)(1.99\times10^{30}\text{ kg})}{(3.00\times10^8 \text{ m/s})^2}$

$= 2.94\times10^{11}\text{ m}$

$\bar{\rho} = \frac{M}{\frac{4}{3}\pi r^3} = \frac{(10^8)(1.99\times10^{30}\text{ kg})}{\frac{4}{3}\pi(2.94\times10^{11}\text{ m})^3}$

$= 1.87\times10^3 \text{ kg/m}^3$

b. $g = \frac{GM}{r^2} = \frac{(6.67\times10^{-11}\frac{\text{m}^3}{\text{kg s}^2})(10^8)(1.99\times10^{30}\text{ kg})}{(2.94\times10^{11}\text{ m})^2}$

$= 1.54\times10^5 \text{ m/s}^2$

c. Values for Sun ($M = 1.99\times10^{30}$ kg, $r = 6.96\times10^8$ m)

$\bar{\rho} = \frac{(1.99\times10^{30}\text{ kg})}{\frac{4}{3}\pi(6.96\times10^8\text{ m})^3} = 1.41\times10^3 \text{ kg/m}^3$

$g = \frac{(6.67\times10^{-11}\frac{\text{m}^3}{\text{kg s}^2})(1.99\times10^{30}\text{ kg})}{(6.96\times10^8\text{ m})^2} = 274 \text{ m/s}^2$

3. a. $$\omega_{max} = \frac{L_{max}}{I} = \frac{\frac{GM^2}{c}}{MR_s^2} = \frac{\frac{\cancel{G}M^{\cancel{2}}}{c}}{M\frac{4G^{\cancel{2}}M^{\cancel{2}}}{c^{\cancel{4}3}}}$$

$$= \frac{c^3}{4GM}$$

$$= \frac{(3.00\times10^8\ \mathrm{m/s})^3}{4(6.67\times10^{-11}\ \frac{\mathrm{m^3}}{\mathrm{kg\,s^2}})(10^8)(1.99\times10^{30}\ \mathrm{kg})}$$

$$= 5.09\times10^{-4}\ \mathrm{s^{-1}}$$

b. $$V = BR^2\omega_{max} = B\,\frac{\cancel{4}G^{\cancel{2}}M^{\cancel{2}}}{c^{\cancel{4}}}\cdot\frac{c^{\cancel{3}}}{\cancel{4}\cancel{G}\cancel{M}}$$

$$= (1\,\mathrm{T})\,\frac{(6.67\times10^{-11}\ \frac{\mathrm{Nm^2}}{\mathrm{kg^2}})(10^8)(1.99\times10^{30}\ \mathrm{kg})}{(3.00\times10^8\ \mathrm{m/s})}$$

$$= 4.42\times10^{19}\ \underbrace{\frac{\mathrm{V}\cdot\cancel{\mathrm{s}}}{\cancel{\mathrm{m^2}}}}_{1\,\mathrm{T}}\cdot\frac{\cancel{\mathrm{m^2}}}{\cancel{\mathrm{s}}}$$

c. $$P = \frac{V^2}{R} = \frac{(4.42\times10^{19}\ \mathrm{V})^2}{30\,\Omega} = 6.51\times10^{37}\ \mathrm{W}$$

4. $\rho_{c,0} = 4.17\times10^{-28}\ \frac{\mathrm{kg}}{\mathrm{m^3}}$ $\quad T = 10^{10}\ \mathrm{K}$ $\quad T_0 = 2.725\ \mathrm{K}$

$$TR = T_0 \Rightarrow R = \frac{T_0}{T} = \frac{2.725\ \mathrm{K}}{10^{10}\ \mathrm{K}} = 2.725\times10^{-10}$$

$$R^3\rho = \rho_0 \Rightarrow \rho_b = \frac{\rho_0}{R^3} = \frac{4.17\times10^{-28}\ \mathrm{kg/m^3}}{(2.725\times10^{-10})^3} = 20.6\ \frac{\mathrm{kg}}{\mathrm{m^3}}$$

5. a. $$\frac{d^2R}{dt^2} = \left\{-\frac{4}{3}\pi G\left[\rho_m + \frac{3P_m}{c^2}\right] + \frac{1}{3}\Lambda c^2\right\}R$$

Static $\Rightarrow \frac{d^2R}{dt^2} = 0$ $\qquad$ Pressureless $\Rightarrow P_m = 0$

$$0 = -\frac{4}{3}\pi G \rho_m + \frac{1}{3}\Lambda c^2$$

$$\Lambda = \frac{4\pi G}{c^2}\rho_m$$

b. $$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho_m - \frac{1}{3}\Lambda c^2\right]R^2 = -kc^2$$

Static $\Rightarrow \frac{dR}{dt} = 0$

$$\left[+\frac{8}{3}\pi G\rho_m + \frac{1}{3}\frac{4\pi G}{\cancel{c^2}}\rho_m \cancel{c^2}\right]R^2 = +kc^2$$

$$\frac{4\pi G}{c^2}\rho_m R^2 = k$$

c. Since $k > 0$, this model is closed.