1. For a spherically symmetric mass distribution with constant density

$$M(r) = \frac{4}{3}\pi r^3 \rho$$

Equating the gravitational force to the centripetal force

$$\frac{GM(r)\cancel{m}}{r^{\cancel{2}}} = \cancel{m}\frac{v^2}{\cancel{r}}$$

$$v^2 = \frac{4}{3}\pi r^2 \rho G$$

$$\omega = \frac{v}{r} = \sqrt{\frac{4}{3}\pi G\rho} = \text{const.} \Rightarrow \text{solid-body rotation}$$

2. M13 $b = 40.9°$ $d = 7\,\text{kpc}$

$$z = d \sin b = (7\,\text{kpc}) \sin 40.9° = 4.6\,\text{kpc}$$

Orion $b = -19.4°$ $d = 450\,\text{pc}$

$$z = (450\,\text{pc}) \sin(-19.4°) = -150\,\text{pc}$$

3. a. $\dot{M} = 10^{-3}\,\frac{M_\odot}{yr}$ $\Delta t = 5\times 10^9\,yr$

$$\Delta M = \dot{M}\Delta t = \left(10^{-3}\,\frac{M_\odot}{yr}\right)\left(5\times 10^{-9}\,yr\right) = 5\times 10^6\,M_\odot$$

b. Very similar to current mass of Sgr A*, suggesting this value for $\dot{M}$ must be close to the historical average.

4. a. For Sa galaxy, Tully-Fisher relation gives

$$M_B = -9.95 \log V_{max} + 3.15$$

For $V_{max} = 324\,\text{km/s}$

$$M_B = -9.95 \log(324) + 3.15 = -21.8$$

b. $m = 12.22$

$$m - M = 5\log(d) - 5$$

$$d = 10^{(m-M+5)/5}$$

$$= 10^{(12.22+21.8+5)/5}$$

$$= 10^{7.8}\,\text{pc} = 6.5\times10^{7}\,\text{pc} = 65\,\text{Mpc}$$

5. For $C=1$

$$r_{max} = \left(\frac{t_{max}\,GM}{2\pi V_m}\right)^{1/2}$$

$$= \left[\frac{\left(1.3\times10^{10}\,\text{yr}\cdot 3.16\times10^{7}\,\frac{\text{s}}{\text{yr}}\right)\left(6.67\times10^{-11}\,\frac{\text{Nm}^2}{\text{kg}^2}\right)\left(5\times10^{6}\,M_\odot\cdot 1.99\times10^{30}\,\frac{\text{kg}}{M_\odot}\right)}{2\pi\left(2.20\times10^{5}\,\text{m/s}\right)}\right]^{1/2}$$

$$= 1.40\times10^{19}\,\text{m} = 454\,\text{pc}$$

6. a. $T_{virial} = 6\times10^{5}\,\text{K}$ $\quad n = 5\times10^{4}\,\text{m}^{-3}$ $\quad \mu = 0.6$ $\quad \rho_0 = \mu m_H n$

$$M_J = \left(\frac{5kT}{G\mu m_H}\right)^{3/2}\left(\frac{3}{4\pi\rho_0}\right)^{1/2}$$

$$= \left[\frac{5\left(1.38\times10^{-23}\,\text{J/K}\right)\left(6\times10^{5}\,\text{K}\right)}{\left(6.67\times10^{-11}\,\frac{\text{Nm}^2}{\text{kg}^2}\right)(0.6)\left(1.67\times10^{-27}\,\text{kg}\right)}\right]^{3/2}\left[\frac{3}{4\pi(0.6)\left(1.67\times10^{-27}\,\text{kg}\right)\left(5\times10^{4}\,\text{m}^{-3}\right)}\right]^{1/2}$$

$$= 1.06\times10^{42}\,\text{kg} = 5.35\times10^{11}\,M_\odot$$

b. Just change T from $6\times10^{5}\,\text{K}$ to $10^{4}\,\text{K}$

$$M_J = 1.15\times10^{9}\,M_\odot = 2.29\times10^{39}\,\text{kg}$$