1. $\Delta T = T_{moving} - T_{rest}$

Using (29.62) $\Delta T \approx T_{rest} \frac{v}{c} \cos\theta$

Take $\cos\theta = 1$, $v = 370.6$ km/s, $T_{rest} = 2.725$ K

$$\Delta T = (2.725\,K)\frac{(370.6\ km/s)}{(3.00 \times 10^5\ km/s)} = 0.00337\ K$$

2. Start from the 1st Law of Thermodynamics (a form of the principle of Conservation of Energy)

$$dU = dQ - dW$$

Co-moving sphere, so no heat flow into or out of volume $(dQ = 0)$

$$\frac{dU}{dt} = \frac{-dW}{dt} = -p\frac{dV}{dt}$$

$$U_{rel} = u_{rel}V = \rho_{rel}c^2\frac{4}{3}\pi r^3 \qquad P_{rel} = \frac{1}{3}\rho_{rel}c^2$$

$$= \rho_{rel}c^2\frac{4}{3}\pi R^3 \varpi^3$$

$$\frac{d}{dt}\left(\rho_{rel}\cancel{c^2}\cancel{\tfrac{4}{3}}\cancel{\pi}R^3\cancel{\varpi^3}\right) = -\frac{1}{3}\rho_{rel}\cancel{c^2}\frac{d}{dt}\left(\cancel{\tfrac{4}{3}}\cancel{\pi}R^3\cancel{\varpi^3}\right)$$

$$\rho_{rel}\frac{d(R^3)}{dt} + R^3\frac{d\rho_{rel}}{dt} = \frac{-1}{3}\rho_{rel}\frac{d(R^3)}{dt}$$

$$R^{\cancel{3}}\frac{d\rho_{rel}}{dt} = \frac{-4}{\cancel{3}}\rho_{rel}\cdot\cancel{3}R^{\cancel{2}}\frac{dR}{dt}$$

$$\int_{\rho_{rel,0}}^{\rho_{rel}}\frac{1}{\rho'_{rel}}\,d\rho'_{rel} = -4\int_1^R \frac{1}{R'}\,dR'$$

$$\ln\rho_{rel} - \ln\rho_{rel,0} = -4\ln R + 4\ln 1$$

$$\frac{\rho_{rel}}{\rho_{rel,0}} = R^{-4} \quad \text{or} \quad R^4 \rho_{rel} = \rho_{rel,0} \quad \text{Q.E.D.}$$

3. $$t(R) = \frac{2}{3} \frac{R_{r,m}^{3/2}}{H_0 \sqrt{\Omega_{m,0}}} \left[2 + \left(\frac{R}{R_{r,m}} - 2 \right) \sqrt{\frac{R}{R_{r,m}} + 1} \right]$$

For $R << R_{r,m}$

$$2 + \left(\frac{R}{R_{r,m}} - 2 \right) \sqrt{\frac{R}{R_{r,m}} + 1} = 2 + \left(\frac{R}{R_{r,m}} - 2 \right) \left[1 + \frac{1}{2} \frac{R}{R_{r,m}} - \frac{1}{8} \left(\frac{R}{R_{r,m}} \right)^2 + \ldots \right]$$

$$= \cancel{2} + \cancel{\frac{R}{R_{r,m}}} - \cancel{2} - \cancel{\frac{R}{R_{r,m}}} + \frac{1}{2} \left(\frac{R}{R_{r,m}} \right)^2 + \frac{1}{4} \left(\frac{R}{R_{r,m}} \right)^2 + \ldots$$

$$= \frac{3}{4} \left(\frac{R}{R_{r,m}} \right)^2$$

Using $R_{r,m} = \dfrac{g_* a T_0^4}{2 \rho_c \Omega_{m,0} c^2}$ and $\rho_c = \dfrac{3 H_0^2}{8 \pi G}$

$$t(R) = \frac{\cancel{2}}{\cancel{3}} \left(\frac{g_* a T_0^4}{2 \rho_c \cancel{\Omega_{m,0}} c^2} \right)^{-1/2} H_0^{-1} \cancel{\Omega_{m,0}^{-1/2}} \frac{\cancel{3}}{\cancel{4} 2} R^2$$

$$= \frac{1}{2} \left(\frac{g_* a T_0^4 \, 8 \pi G}{6 \cancel{H_0^2} c^2} \right)^{-1/2} \cancel{H_0^{-1}} R^2$$

$$\Rightarrow \quad R(t) = \left(\frac{16 \pi G g_* a}{3 c^2} \right)^{1/4} T_0 \, t^{1/2}$$

4. Evaluate $n_p \sigma v \Delta t$

$$n_p = \frac{\rho_b}{m_p} \qquad \rho_b = \rho_{b,0} (1+z)^3 \qquad \rho_{b,0} = 4.17 \times 10^{-28} \frac{kg}{m^3}$$

$$\sigma = \pi (2r)^2 \qquad r = 10^{-15} \, m$$

$$v = \sqrt{\frac{3kT}{m_n}} \qquad T = 10^9 K \qquad T_0 = 2.725 \, K$$

ASTR312
HW#9
3

$R = \frac{T_0}{T} = 2.725 \times 10^{-9}$ $z = \frac{1}{R} - 1 = 3.67 \times 10^{8}$

Eq. (29.89) $\Delta t = \left[\frac{1.81 \times 10^{10} \ K s^{1/2} \ g_*^{-1/4}}{T} \right]^2$ Take $g_* = 3.363$

$n_p = 1.2 \times 10^{25} \ m^{-3}$ $\sigma = 1.3 \times 10^{-29} \ m^2$

$v = 5.0 \times 10^6 \ m/s$ $\Delta t = 179 \ s$

$n_p \sigma v \Delta t = 1.4 \times 10^5 \gg 1$