1.a. Using $R_{flat} = \frac{1}{1+z}$ to replace R_{flat} in (29.31), we have

$$\frac{1}{1+z} = \left(\frac{3}{2}\right)^{2/3}\left(\frac{t}{t_H}\right)^{2/3}$$

Solving for $\frac{t}{t_H}$, we get

$$\frac{t}{t_H} = \frac{2}{3}\left(\frac{1}{1+z}\right)^{3/2}$$ which is (29.40)

b. For $z=0$, $t = \frac{2}{3}t_H$ (for flat universe)

c. $t_H = \frac{1}{H_0} = \frac{1}{(3.24\times10^{-18})(0.70)}\,s = 4.41\times10^{17}\,s = 14.0\ Gyr$

$$t_{0,flat} = \frac{2}{3}t_H = 9.30\ Gyr$$

d. This age is close to, though a bit smaller than, the ages of the oldest globular clusters and the accepted age of the Universe. Since this very simple model only considered a Universe made of pressureless dust, it is not surprising that the result isn't closer to the true value.

2. $$R_{open} = \frac{4\pi G\rho_0}{3|k|c^2}\left[\cosh(x)-1\right]$$

$$t_{open} = \frac{4\pi G\rho_0}{3|k|^{3/2}c^3}\left[\sinh(x)-x\right]$$

Using $\frac{dR}{dt} = \frac{\frac{\partial R}{\partial x}dx}{\frac{\partial t}{\partial x}dx}$

$$\frac{\partial R}{\partial x} = \frac{4\pi G\rho_0}{3|k|c^2}\sinh(x) \qquad \frac{\partial t}{\partial x} = \frac{4\pi G\rho_0}{3|k|^{3/2}c^3}\left[\cosh(x)-1\right]$$

$$\frac{dR}{dt} = |k|^{1/2}c\,\frac{\sinh(x)}{\left[\cosh(x)-1\right]}$$

Plugging in to (29.11)

$$\left(\frac{dR}{dt}\right)^2 - \frac{8\pi G\rho_0}{3R} = -kc^2$$

$$|k|c^2\frac{\sinh^2(x)}{[\cosh(x)-1]^2} - \frac{8\pi G\rho_0}{3\left[\frac{4\pi G\rho_0}{3|k|c^2}(\cosh(x)-1)\right]} = -kc^2$$

$$\frac{\sinh^2(x) - 2[\cosh(x)-1]}{[\cosh(x)-1]^2} = \frac{-k}{|k|}$$

Since $k<0$ $\quad \frac{-k}{|k|} = 1$

$$\frac{\cosh^2(x) - 1 - 2\cosh(x) + 2}{[\cosh(x)-1]^2} = 1$$

$$\frac{[\cosh(x)-1]^2}{[\cosh(x)-1]^2} = 1$$

$$1 = 1 \qquad \text{Q.E.D.}$$

3. $T_{cloud} = 7.4 \pm 0.8\ K \qquad z = 1.776 \qquad R = \frac{1}{1+z} = 0.360$

$TR = T_0 \qquad T_0 = 2.725\ K$

$$T = \frac{T_0}{R} = \frac{2.725\ K}{0.360} = 7.57\ K$$

The temperature of the cloud is consistent with the temperature of the CMB at that redshift.