Homework #6
Solutions
①

#1. Problem 28.3

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In[19]:= T = 7.3*^5
Out[19]= 730000.

In[20]:= h = 6.626*^-34
Out[20]= 6.626 × 10^-34

In[21]:= c = 2.9979*^8
Out[21]= 2.9979 × 10^8

In[22]:= k = 1.3807*^-23
Out[22]= 1.3807 × 10^-23

In[28]:= Planck = Log[10, (2.0 * h * nu^4 / c^2 / (Exp[h * nu / (k * T)] - 1.))]
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Out[28]= $\dfrac{\mathrm{Log}\left[\dfrac{1.47451\times10^{-50}\,10^{4x}}{-1.+e^{6.57399\times10^{-17}\,10^{x}}}\right]}{\mathrm{Log}[10]}$

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In[17]:= nu = 10^x
Out[17]= 10^x

In[29]:= Plot[Planck, {x, 15.5, 17.5}]
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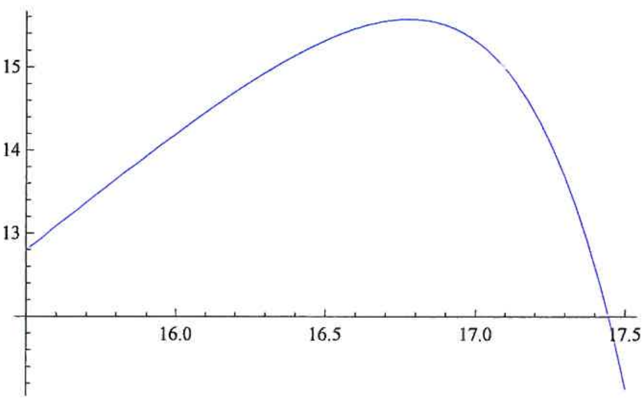
Out[29]=



#6
Homework [redacted]
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[redacted] ②

[redacted]

2. $$z = \left[\frac{\left(1+\frac{v}{c}\right)}{\left(1-\frac{v}{c}\right)}\right]^{1/2} - 1$$

$$\frac{1+\frac{v}{c}}{1-\frac{v}{c}} = (z+1)^2$$

$$1+\frac{v}{c} = (z+1)^2 - \frac{v}{c}(z+1)^2$$

$$\frac{v}{c}\left[1+(z+1)^2\right] = (z+1)^2 - 1$$

$$\frac{v}{c} = \frac{(z+1)^2-1}{(z+1)^2+1}$$

The denominator will always be larger than the numerator.

$\therefore$ $\frac{v}{c}$ will always be less than 1

3. $$z = \frac{\Delta\lambda}{\lambda_0} = \frac{\lambda-\lambda_0}{\lambda_0} = \frac{\frac{1}{f}-\frac{1}{f_0}}{\frac{1}{f_0}} = 0.170$$

a. $$z\frac{1}{f_0} + \frac{1}{f_0} = \frac{1}{f}$$

$$\frac{1}{f_0} = \frac{1}{f}\,\frac{1}{z+1} \qquad f_0 = f(z+1)$$

$$f_0 = 10^3\,\text{MHz}\,(0.170+1) = 1.17\times 10^3\,\text{MHz}$$

b. $\delta L = 4\pi d^2 \delta F$

$$= 4\pi (240\times10^6 \text{ pc})^2 \left(\frac{3.09\times10^{16}\text{ m}}{\text{pc}}\right)^2 \left(2.18\times10^{-30}\frac{\text{erg}}{\text{m}^2\text{ s Hz}}\right)$$

$$= 1.51\times10^{21}\frac{\text{erg}}{\text{s}\cdot\text{Hz}} = 1.51\times10^{28}\frac{\text{J}}{\text{s}\cdot\text{Hz}}$$

c. $L = \Delta\nu\,\delta L = (10^4\text{ Hz})\left(1.51\times10^{28}\frac{\text{J}}{\text{s}\cdot\text{Hz}}\right) = 1.51\times10^{32}\text{ W}$

d. $$\frac{26.73\times10^6\text{ eV}}{4\text{ H}}\cdot\frac{1.60\times10^{-19}\text{ J}}{\text{eV}} = 1.07\times10^{-12}\frac{\text{J}}{\text{H}}$$

$$\frac{1.51\times10^{32}\text{ W}}{1.07\times10^{-12}\text{ J/H}} = 1.41\times10^{44}\text{ H/s}$$

$$1.41\times10^{44}\frac{\text{H}}{\text{s}}\cdot1.67\times10^{-27}\frac{\text{kg}}{\text{H}} = 2.35\times10^{17}\text{ kg/s}$$

e. $$2.35\times10^{17}\frac{\text{kg}}{\text{s}}\cdot10^8\text{ yrs}\left(3.16\times10^7\frac{\text{s}}{\text{yr}}\right)\left(\frac{1\,M_\odot}{1.99\times10^{30}\text{ kg}}\right)$$

$= 374\,M_\odot$ Unlikely to be a "stellar" source