Astronomy 312 - Fragile
Homework 5 (assigned 2/11/16; due 2/23/16)

(2) 1. (a) By equating the cooling timescale to the free-fall timescale, show that the maximum mass of a protogalactic nebular is given by

$$M = \frac{25}{32}\frac{\Lambda^2}{G^3\mu^4 m_H^4 R} \ . \tag{1}$$

(2) (b) Estimate the maximum mass of a protogalactic nebular that can undergo a free-fall collapse if $R = 60$ kpc. Assume that $\Lambda \simeq 10^{-37}$ W m^3.

2. The Large Magellanic Cloud, which has a mass of about $2 \times 10^{10} M_\odot$, orbits the Milky Way at a distance of 51 kpc.

(2) (a) Estimate the tidal radius of the Large Magellanic Cloud as it orbits the Milky Way. Take the mass of the Milky Way's dark-matter halo to be $5.4 \times 10^{11} M_\odot$.

(1) (b) How does your answer compare with the size of the LMC, which has an angular diameter of $460'$?

3. Assume an initial mass function of the form

$$\xi(M) = \frac{dN}{dM} = CM^{-(1+x)} \ . \tag{2}$$

(2) (a) If $x = 1.8$, calculate the ratio of the number of stars that are formed in the mass range between $2M_\odot$ and $3M_\odot$ to those formed with masses between $10M_\odot$ and $11M_\odot$.

(2) (b) Beginning with the initial mass function and using the mass-luminosity relation for main-sequence stars,

$$\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^\alpha \ , \tag{3}$$

derive an expression for the number of main-sequence stars formed per unit luminosity interval, dN/dL.

(2) (c) If $x = 1.8$ and $\alpha = 4$, calculate the ratio of the number of stars that are formed with main-sequence luminosities between $2L_\odot$ and $3L_\odot$ to the number of those formed with luminosities between $10L_\odot$ and $11L_\odot$.

(1) (d) Compare your answers in parts (a) and (c) and explain what the results say about the physical characteristics of stars along the main sequence.