1. $$\frac{d^2\vec{r}}{dt^2} = \frac{d}{dt}\left(\frac{d}{dt}\left(R\hat{e}_R + z\hat{e}_z\right)\right)$$

$$= \frac{d}{dt}\left(\frac{d}{dt}\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\right) \qquad x = R\cos\phi$$

$$= \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k} \qquad y = R\sin\phi$$

$$= \frac{d}{dt}\left(\frac{d}{dt}R\cos\phi\right)\hat{i} + \frac{d}{dt}\left(\frac{d}{dt}R\sin\phi\right)\hat{j} + \ddot{z}\hat{k}$$

$$= \frac{d}{dt}\left(\dot{R}\cos\phi - R\dot{\phi}\sin\phi\right)\hat{i} + \frac{d}{dt}\left(\dot{R}\sin\phi + R\dot{\phi}\cos\phi\right)\hat{j} + \ddot{z}\hat{k}$$

$$= \left[\ddot{R}\cos\phi - \dot{R}\dot{\phi}\sin\phi - \dot{R}\dot{\phi}\sin\phi - R\ddot{\phi}\sin\phi - R\dot{\phi}^2\cos\phi\right]\hat{i}$$
$$+ \left[\ddot{R}\sin\phi + \dot{R}\dot{\phi}\cos\phi + \dot{R}\dot{\phi}\cos\phi + R\ddot{\phi}\cos\phi - R\dot{\phi}^2\sin\phi\right]\hat{j} + \ddot{z}\hat{k}$$

$$= (\ddot{R} - R\dot{\phi}^2)(\cos\phi\,\hat{i} + \sin\phi\,\hat{j}) + (2\dot{R}\dot{\phi} + R\ddot{\phi})(-\sin\phi\,\hat{i} + \cos\phi\,\hat{j}) + \ddot{z}\hat{k}$$

$$= (\ddot{R} - R\dot{\phi}^2)\hat{e}_R + (2\dot{R}\dot{\phi} + R\ddot{\phi})\hat{e}_\phi + \ddot{z}\hat{e}_z$$

$$= (\ddot{R} - R\dot{\phi}^2)\hat{e}_R + \frac{1}{R}\frac{\partial(R^2\dot{\phi})}{\partial t}\hat{e}_\phi + \ddot{z}\hat{e}_z$$

2.a. $\rho(t) = A_R \sin \kappa_0 t$

$\dot{\rho}(t) = A_R \kappa_0 \cos \kappa_0 t$

$\kappa_0 = 35.6 \frac{km}{s\ kpc} \qquad \dot{\rho} = -9\ km/s$

If Sun is currently passing through its equilibrium, then $\rho = 0$. Take $t = 0$.

$$A_R = \frac{\dot{\rho}}{\kappa_0} = \frac{-9\ \frac{km}{s}}{35.6\ \frac{km}{s\ kpc}} = 0.25\ kpc = 250 pc$$

b. This estimate is a minimum because, if the Sun isn't currently passing through its equilibrium, then we underestimated $\dot{\rho}(0)$.

3. a. $\rho = 0.05 \frac{M_\odot}{pc^3}$ M0 star: $M = 0.51 M_\odot$ $R = 0.63 R_\odot$

Number density of stars $= n = \frac{\rho}{M} = \frac{0.05 \frac{M_\odot}{pc^3}}{0.51 M_\odot}$

$= 0.098 \frac{stars}{pc^3}$

Volume of space per star $= V = \frac{1}{n} = 10.2\ pc^3$

Volume of single star $= V_{star} = \frac{4}{3}\pi R^3$

$= \frac{4}{3}\pi \left(0.63 \cdot 6.96\times10^8 m \cdot \frac{1\ pc}{3.09\times10^{16} m}\right)^3$

$= 1.20\times10^{-23}\ pc^3$

$f = \frac{V_{star}}{V} = \frac{1.20\times10^{-23}\ pc^3}{10.2\ pc^3} = 1.17\times10^{-24}$

b. Volume traced out by intruder star $= V_{int} = \pi R^2 z$

$= \pi \left(0.63 \cdot 6.96\times10^8 m \frac{1pc}{3.09\times10^{16} m}\right)^2 (1000 pc)$

$= 6.33\times10^{-13}\ pc^3$

Probability of intersecting a star in this volume $=$

$\frac{V_{int}}{V} = \frac{6.33\times10^{-13}\ \cancel{pc^3}}{10.2\ \cancel{pc^3}} = 6.20\times10^{-14}$