1. Barnard's Star

$V_r = -108\ \mathrm{km/s}$ $\quad \mu = 10.34''\ \mathrm{yr}^{-1}$ $\quad \pi = 0.546''$

a. $d = \frac{1}{\pi} = \frac{1}{0.546}\ \mathrm{pc} = 1.83\mathrm{pc}$

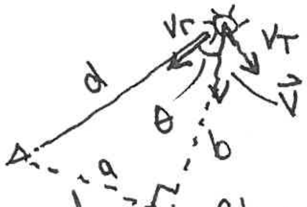
$$= 1.83\mathrm{pc}\left(\frac{3.086\times10^{13}\ \mathrm{km}}{1\mathrm{pc}}\right) = 5.65\times10^{13}\ \mathrm{km}$$

b. $V_T\left(\frac{\mathrm{km}}{\mathrm{s}}\right) = 4.74\, d(\mathrm{pc})\, \mu\left(\frac{''}{\mathrm{yr}}\right)$

$$= 4.74(1.83\ \mathrm{pc})(10.34''/\mathrm{yr}) = 89.7\ \mathrm{km/s}$$

c. $|\vec{V}| = \sqrt{V_r^2 + V_T^2} = \sqrt{(-108\ \mathrm{km/s})^2 + (89.7\ \mathrm{km/s})^2}$

$$= 1.40\times10^2\ \mathrm{km/s}$$



$\theta = \tan^{-1}\left(\frac{89.7\ \mathrm{km/s}}{108\ \mathrm{km/s}}\right) = 39.8°$ w.r.t. line of sight

d. Closest approach (distance a) after Barnard's Star has travelled distance b along current trajectory.

$$b = d\cos\theta = (5.65\times10^{13}\ \mathrm{km})\cos(39.8°)$$
$$= 4.34\times10^{13}\ \mathrm{km}$$

$$t = \frac{b}{V} = \frac{4.34\times10^{13}\ \mathrm{km}}{1.40\times10^2\ \mathrm{km/s}} = 3.088\times10^{11}\mathrm{s} = 9.79\times10^3\ \mathrm{yrs}$$

e. $a = d\sin\theta = (5.65\times10^{13}\ \mathrm{km})\sin(39.8°)$ $\qquad$ α-Centauri

$= 3.62\times10^{13}\ \mathrm{km} = 1.17\mathrm{pc}$ $\qquad$ $d = 1.35\mathrm{pc}$

2. $\sin b = \sin\delta_{NGP}\sin\delta + \cos\delta_{NGP}\cos\delta\cos(\alpha - \alpha_{NGP})$

$$= \sin(27°7'41.7'')\sin\delta + \cos(27°7'41.7'')\cos\delta\cos(\alpha - 12^h51^m26.28^s)$$

$$= \sin(0.473)\sin\delta + \cos(0.473)\cos\delta\cos(\alpha - 3.37)$$

$\alpha_{vega} = 18^h 36^m 56.34^s = 4.87 \text{ rad}$

$\delta_{vega} = +38° 47' 1.3'' = 0.677 \text{ rad}$

$$b = \sin^{-1}\left[\sin(0.473)\sin(0.677) + \cos(0.473)\cos(0.677)\cos(4.87-3.37)\right]$$

$$= 0.341 \text{ rad} = 19.5°$$

$$l = l_{NCP} - \sin^{-1}\left[\frac{\cos\delta \sin(\alpha - \alpha_{NGP})}{\cos b}\right]$$

$$= 123° 55' 55.2'' - \sin^{-1}\left[\frac{\cos\delta \sin(\alpha - 12^h 51^m 26.28^s)}{\cos b}\right]$$

$$= 2.16 - \sin^{-1}\left[\frac{\cos\delta \sin(\alpha - 3.37)}{\cos b}\right]$$

$$= 2.16 - \sin^{-1}\left[\frac{\cos(0.677)\sin(4.87-3.37)}{\cos(0.341)}\right]$$

$$= 1.19 \text{ rad} = 68.2°$$

3. a. $$\rho_{min} = \frac{M}{\frac{4}{3}\pi r_{max}^3} = \frac{M}{\frac{4}{3}\pi r_p^3}$$

$r_{max} = r_p$ for S2 $= 1.8 \times 10^{13}$ m

$$= \frac{(3.7\times10^6 M_\odot)(1.99\times10^{30} \frac{kg}{M_\odot})}{\frac{4}{3}\pi(1.8\times10^{13} m)^3}$$

$$= 3.01\times10^{-4} \text{ kg/m}^3$$

b. $$\rho = \frac{(4.1\times10^6 M_\odot)(1.99\times10^{30} \frac{kg}{M_\odot})}{\frac{4}{3}\pi(1 AU \cdot 1.496\times10^{11} \frac{m}{AU})^3}$$

$$= 582 \frac{kg}{m^3}$$