1. a. Centripetal force = force of gravity

$$\cancel{m}\frac{v^2}{r} = \frac{GM\cancel{m}}{r^2}$$

$$M = \frac{v^2 r}{G} = \frac{(2.2\times10^5\ m/s)^2(8.5\times10^3\ pc)\left(\frac{3.09\times10^{16}\ m}{pc}\right)}{6.67\times10^{-11}\ \frac{N\cdot m^2}{kg^2}}$$

$$= 1.91\times10^{41}\ kg = 9.58\times10^{10}\ M_\odot$$

b. $$P = \frac{2\pi}{\omega} = \frac{2\pi r}{v} = \frac{2\pi(8.5\times10^3\ pc)\left(3.09\times10^{16}\ \frac{m}{pc}\right)}{2.2\times10^5\ m/s}$$

$$= 7.50\times10^{15}\ s = 2.37\times10^8\ yr$$

c. # of orbits = $\frac{\text{age}}{\text{period}} = \frac{4.6\times10^9\ yr}{2.37\times10^8\ yr} = 19.$ times

2. a. $d = 10^{(V - M_V - A_V + 5)/5}$ where A_V is the interstellar extinction

a. $A_V = V - M_V + 5 - 5\log_{10} d(pc)$

$= 13.0 - (-4.15) + 5 - 5\log_{10}(9.0\times10^3)$

$= 2.38$ magnitudes

b. $\frac{2.38\ \text{magnitudes}}{9.0\ kpc} = 0.26\ \frac{\text{magnitudes}}{kpc}$

3. a. Neutral gas in disk consists of

# of atoms = $\frac{(0.5\times10^{10}\ M_\odot)\left(1.99\times10^{30}\ \frac{kg}{M_\odot}\right)}{1.67\times10^{-27}\ kg} = 6\times10^{66}$ atoms of H

Each atom has $\frac{3}{2}kT$ of thermal energy on average

$E_{total} = (6\times10^{66})\frac{3}{2}\left(1.38\times10^{-23}\ \frac{J}{K}\right)(15\ K) = 1.85\times10^{45}\ J$

To get the energy density, divide total thermal energy by volume.

$$V = \pi R^2 h = \pi\left(8\times10^3 \text{ pc} \cdot \frac{3.09\times10^{16} \text{ m}}{\text{pc}}\right)^2\left(160 \text{ pc} \cdot \frac{3.09\times10^{16} \text{ m}}{\text{pc}}\right)$$

$$= 9.5\times10^{59} \text{ m}^3$$

$$u_{thermal} = \frac{1.85\times10^{45} \text{ J}}{9.5\times10^{59} \text{ m}^3} = 1.9\times10^{-15} \text{ J/m}^3$$

b. $$u_m = \frac{B^2}{2\mu_0} = \frac{(0.4\times10^{-9} \text{ T})^2}{2\left(4\pi\times10^{-7} \frac{\text{N}}{\text{A}^2}\right)} = 6.4\times10^{-14} \frac{\text{J}}{\text{m}^3}$$

c. Yes. More energy per unit volume in magnetic field than in thermal energy of gas.